Introduction to computer vision II

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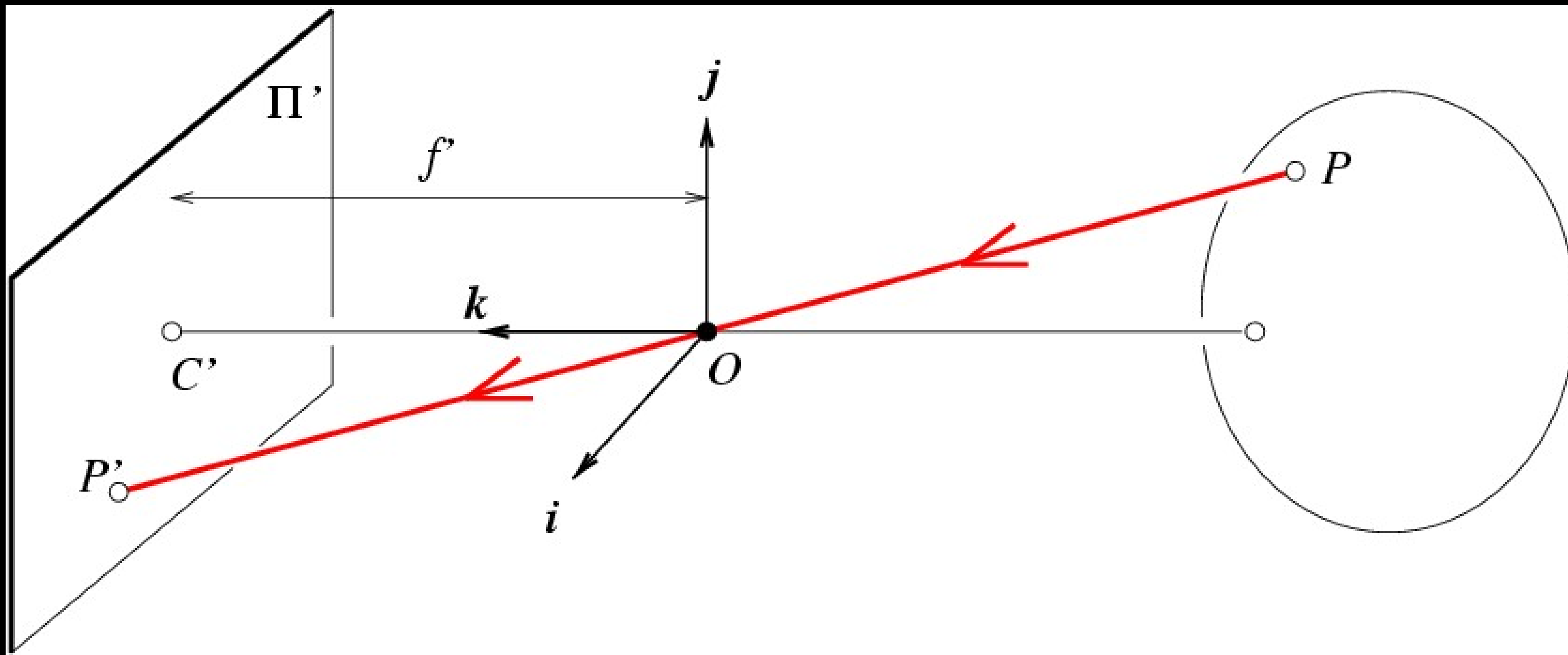
Slides will be available after class at:
https://mtrager.github.io/introCV-fall2019/

Camera geometry and calibration II

- Pinhole perspective projection
- A detour through sensing country
- Intrinsic and extrinsic parameters
- Strong (Euclidean) calibration
- Degenerate configurations
- What about affine cameras?

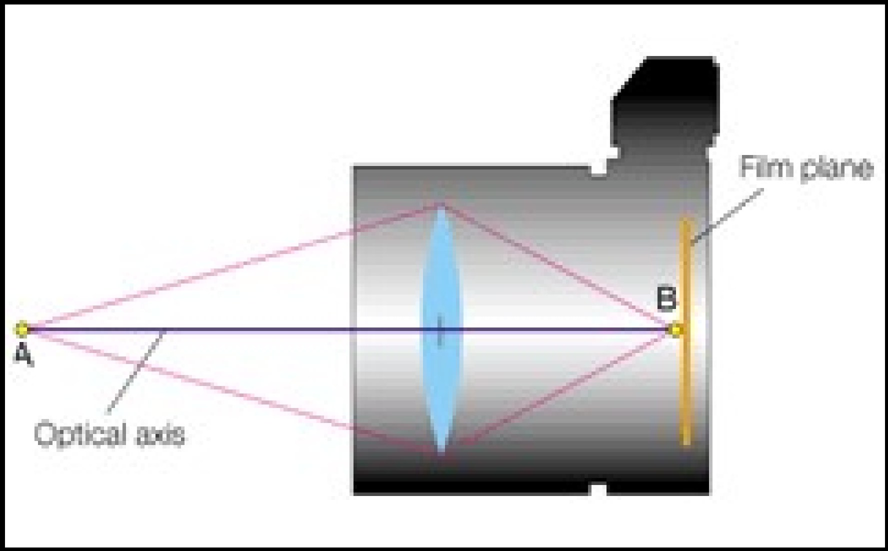
Pinhole Perspective Equation



$$\begin{cases} x' = f'\dfrac{x}{z} \\ y' = f'\dfrac{y}{z} \end{cases}$$

NOTE: *z* is always negative..

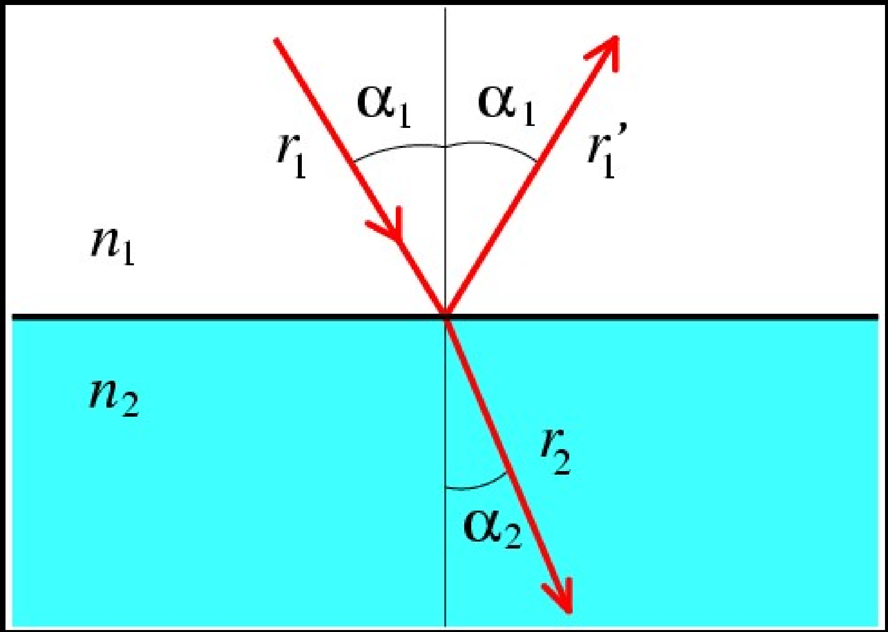
Lenses



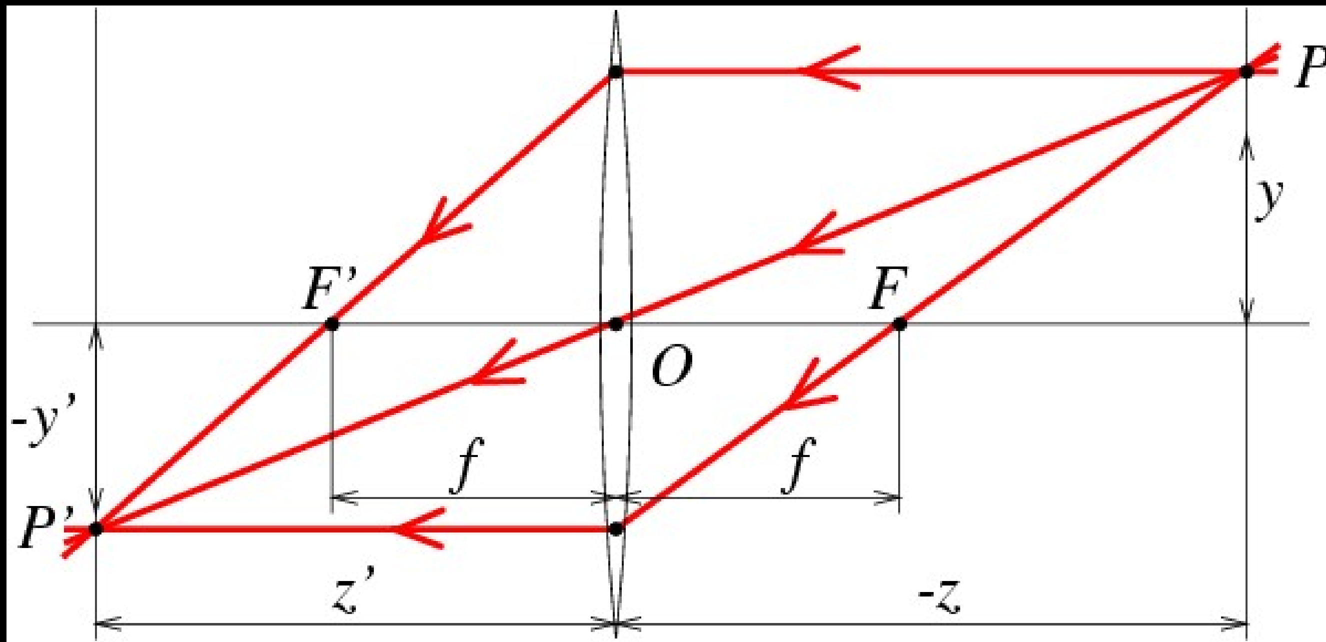
Snell's law

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

(Descartes' law
for Frenchies)

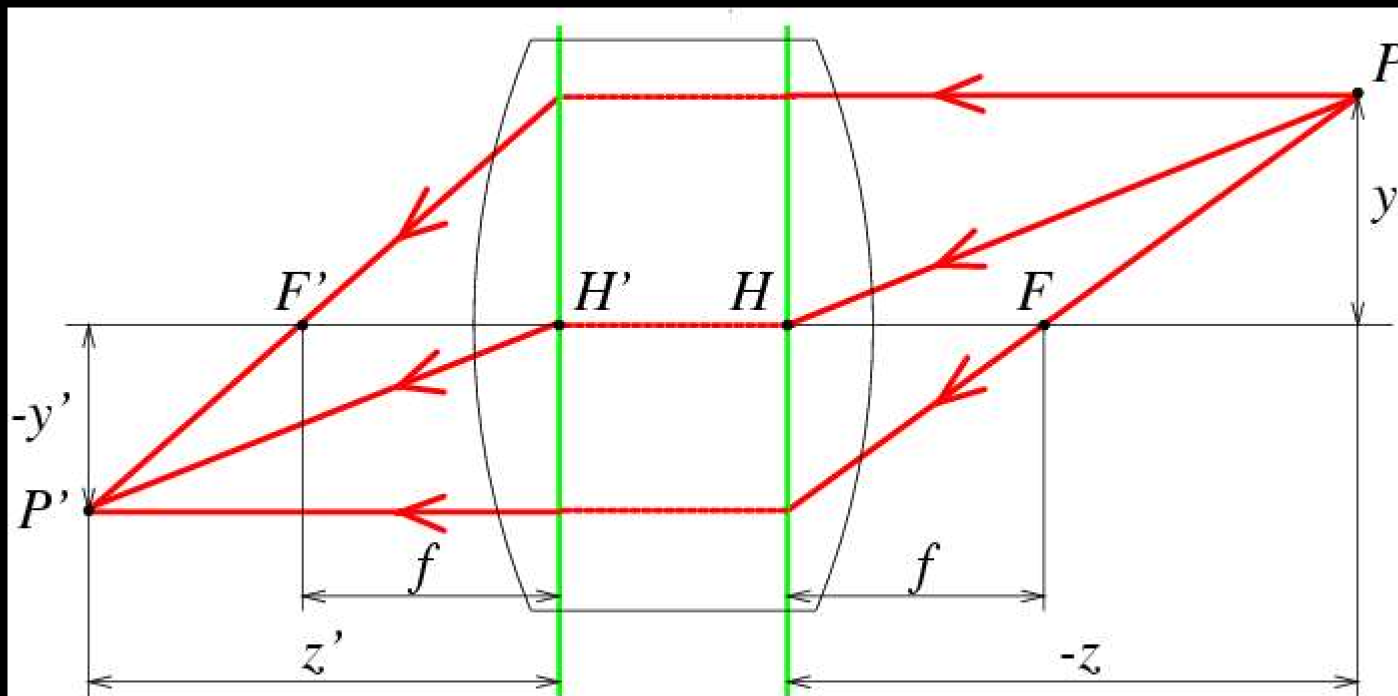


Thin Lenses (including paraxial approximation)

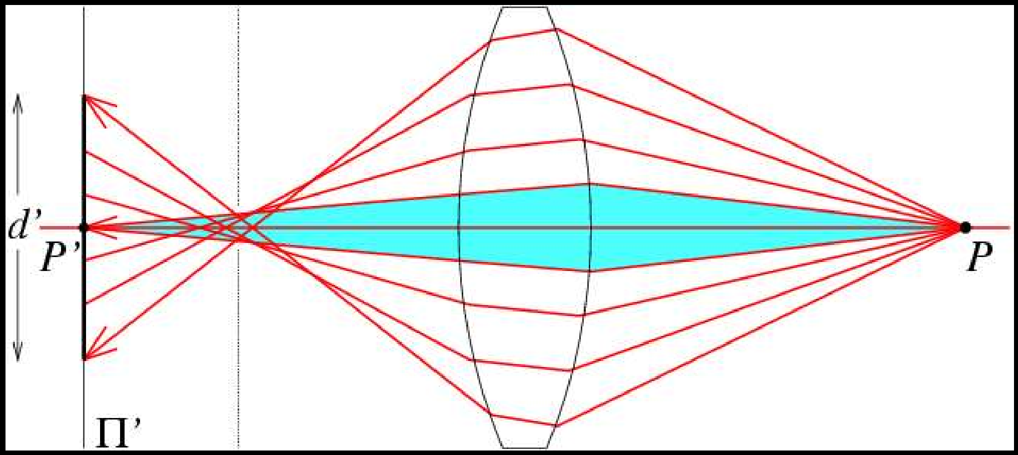


$$\begin{cases} x' = z'\dfrac{x}{z} \\ y' = z'\dfrac{y}{z} \end{cases}$$ where $$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$ and $$f = \frac{R}{2(n-1)}$$

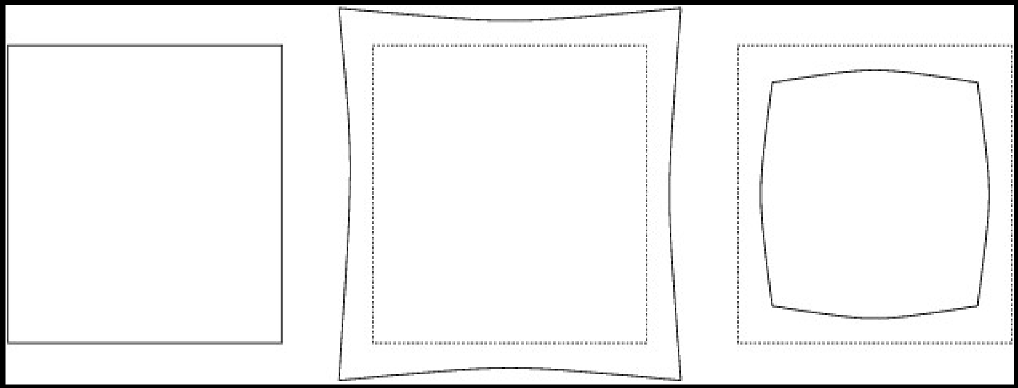
Thick Lenses



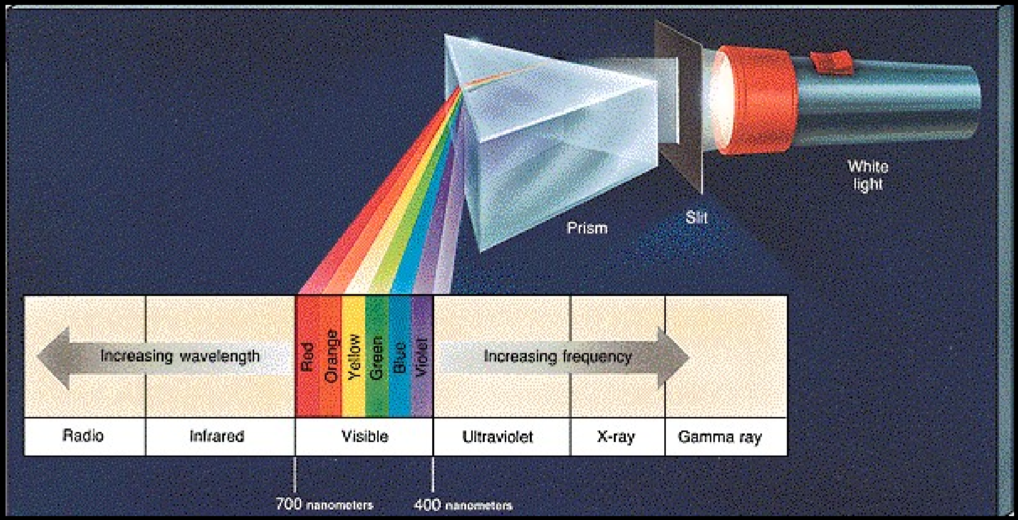
Spherical Aberration



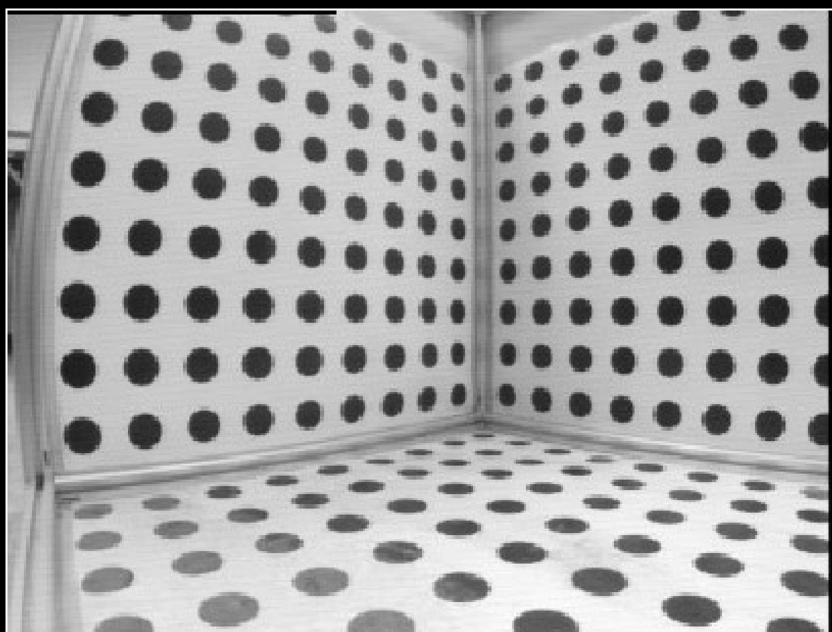
Distortion

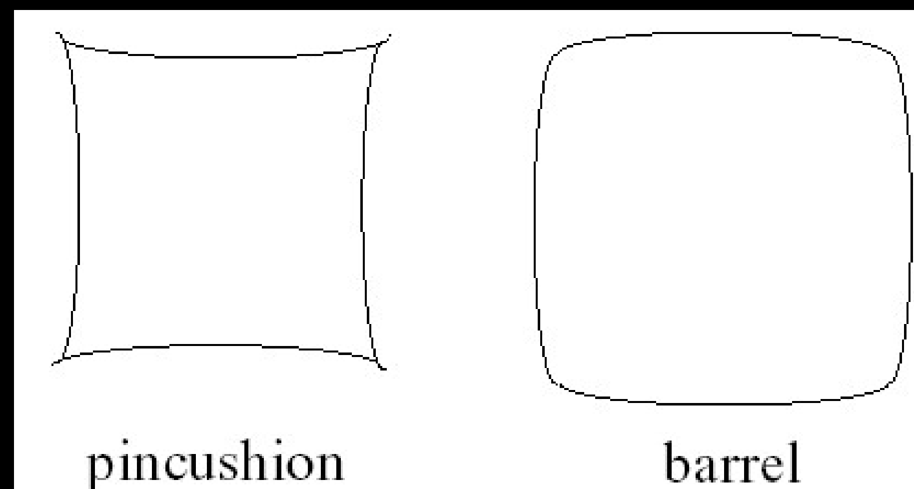


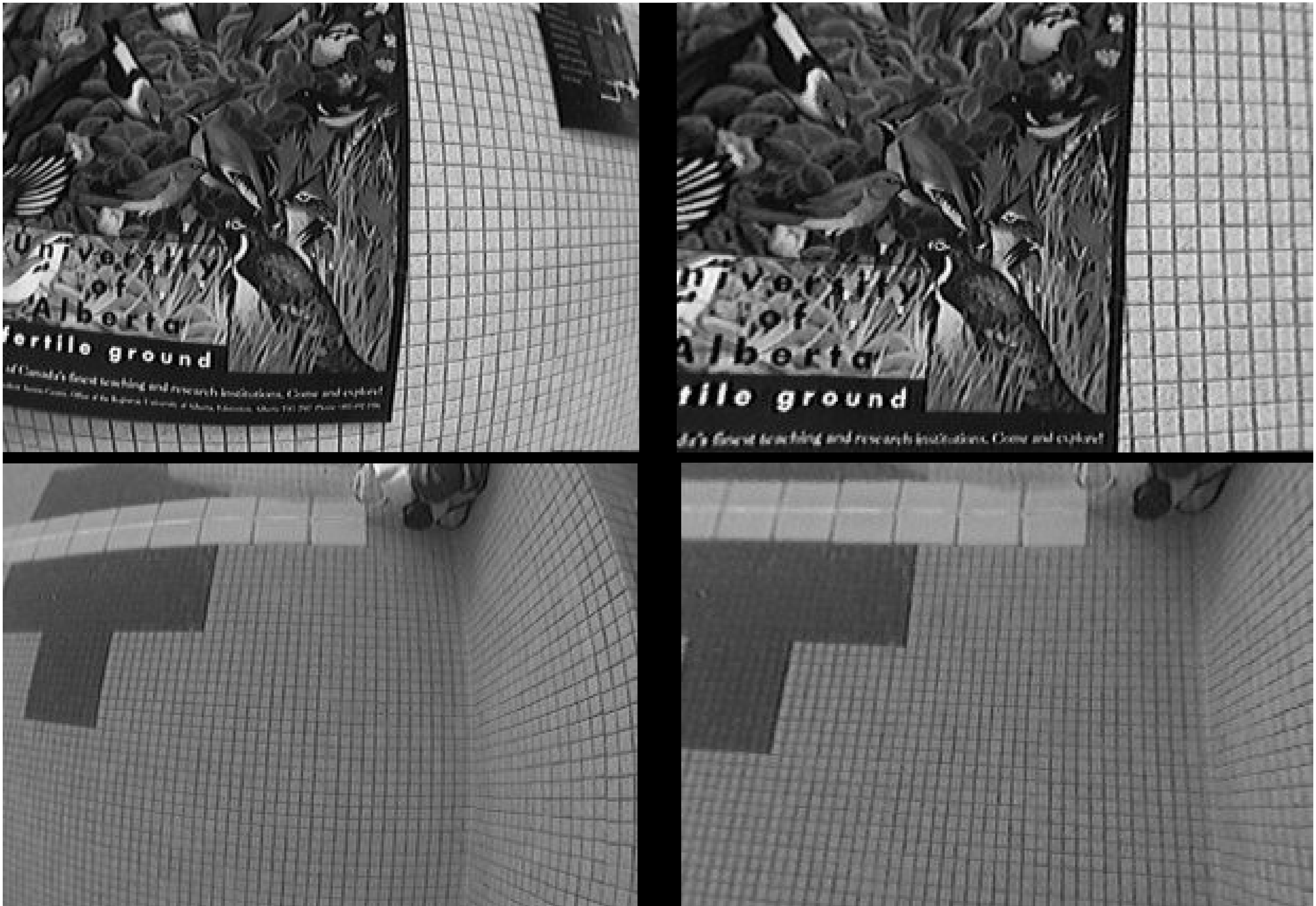
Chromatic Aberration



Geometric Distortion

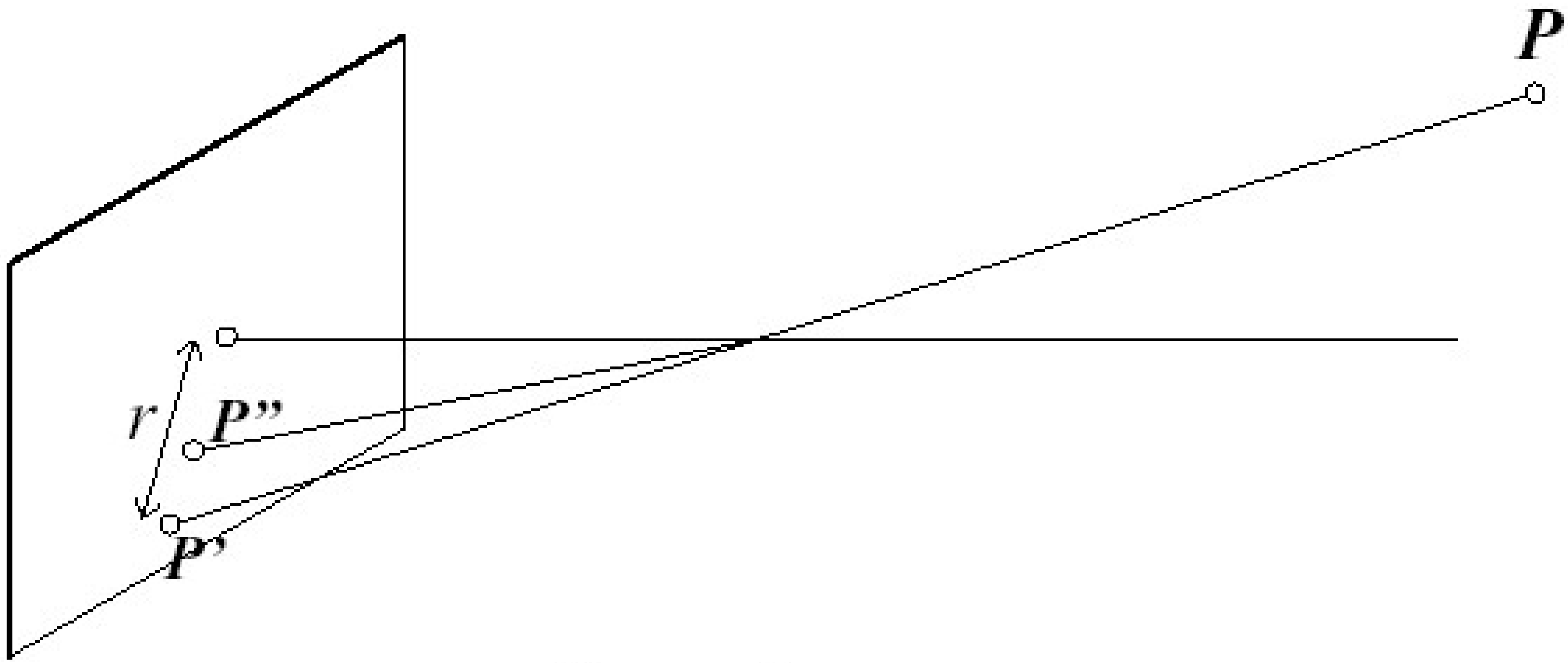






Rectification

Radial Distortion Model



Ideal:

$$x' = f\frac{x}{z}$$

$$y' = f\frac{y}{z}$$

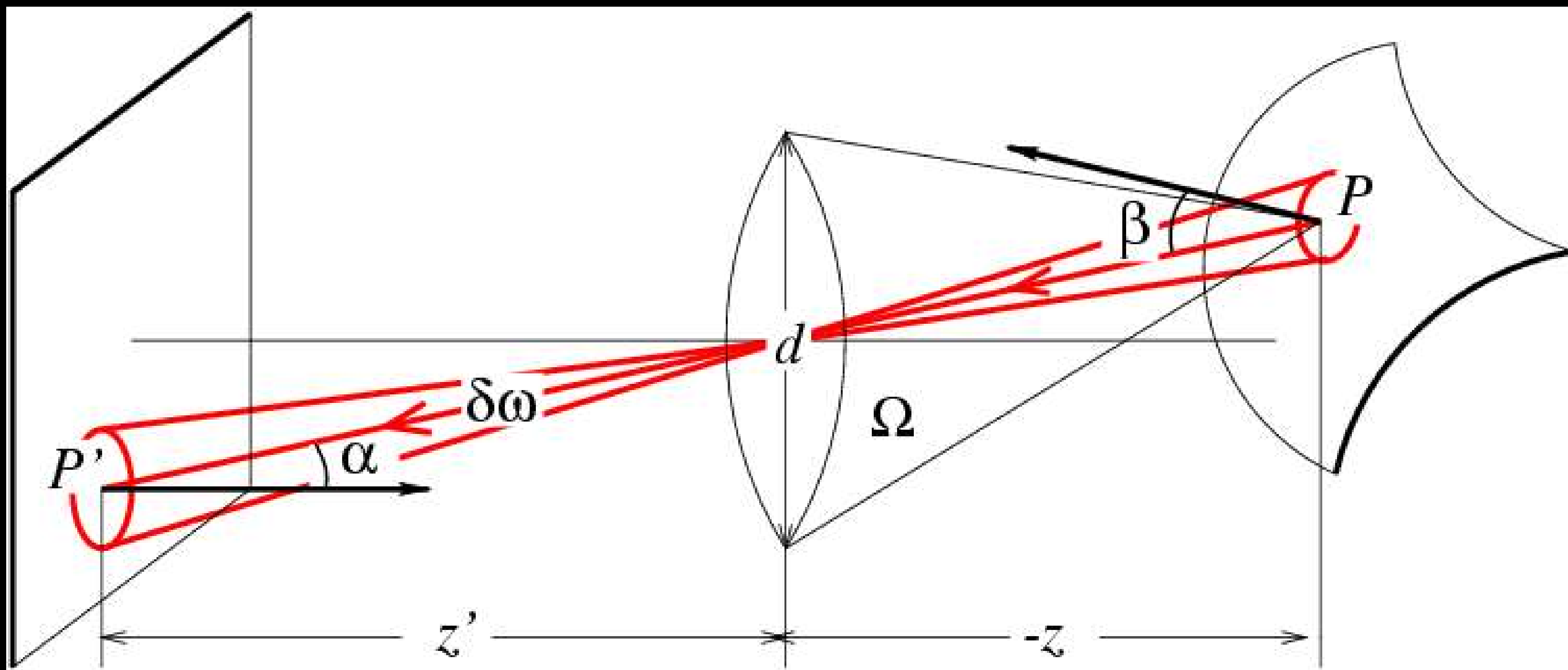
Distorted:

$$x'' = \frac{1}{\lambda}x'$$

$$y'' = \frac{1}{\lambda}y'$$

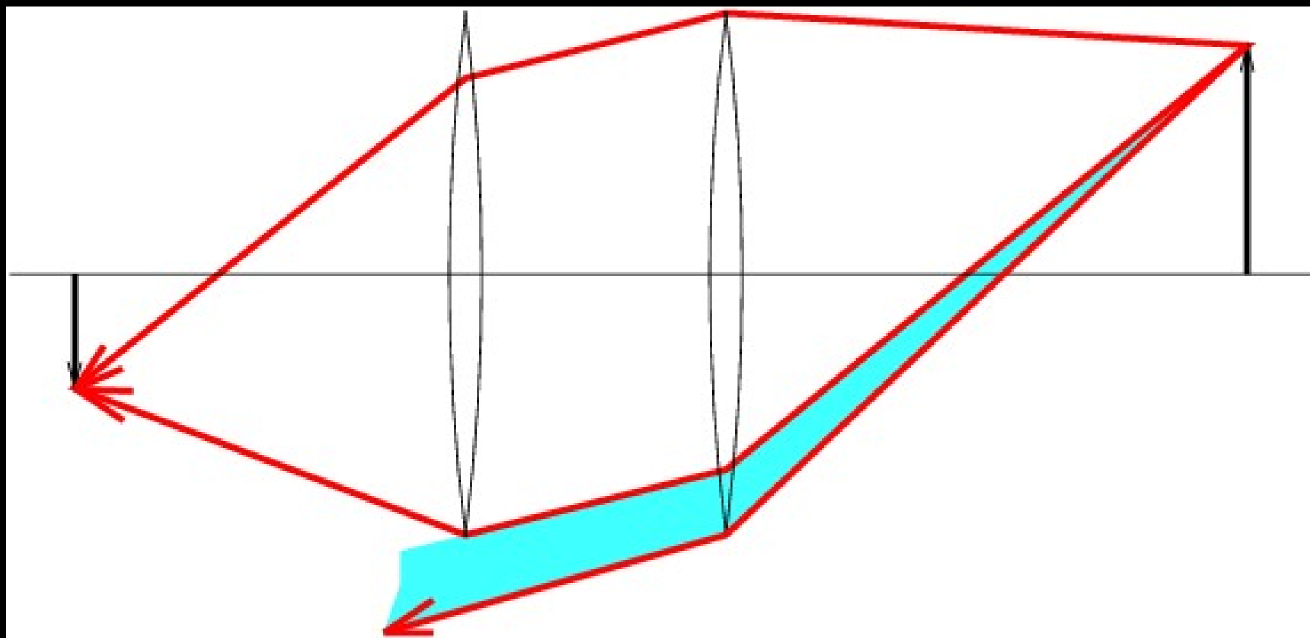
$$\lambda = 1 + k_1 r^2 + k_2 r^4 + \cdots$$

A compound lens



$$E=(\Pi/4)\left[\,(d/z')^2 \cos^4\alpha\,\right] L$$

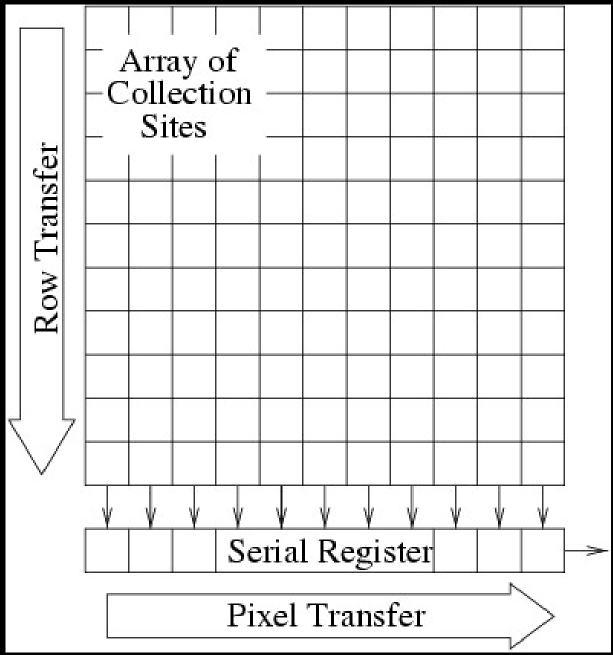
Vignetting




Challenge: Illumination – What is wrong with these pictures?

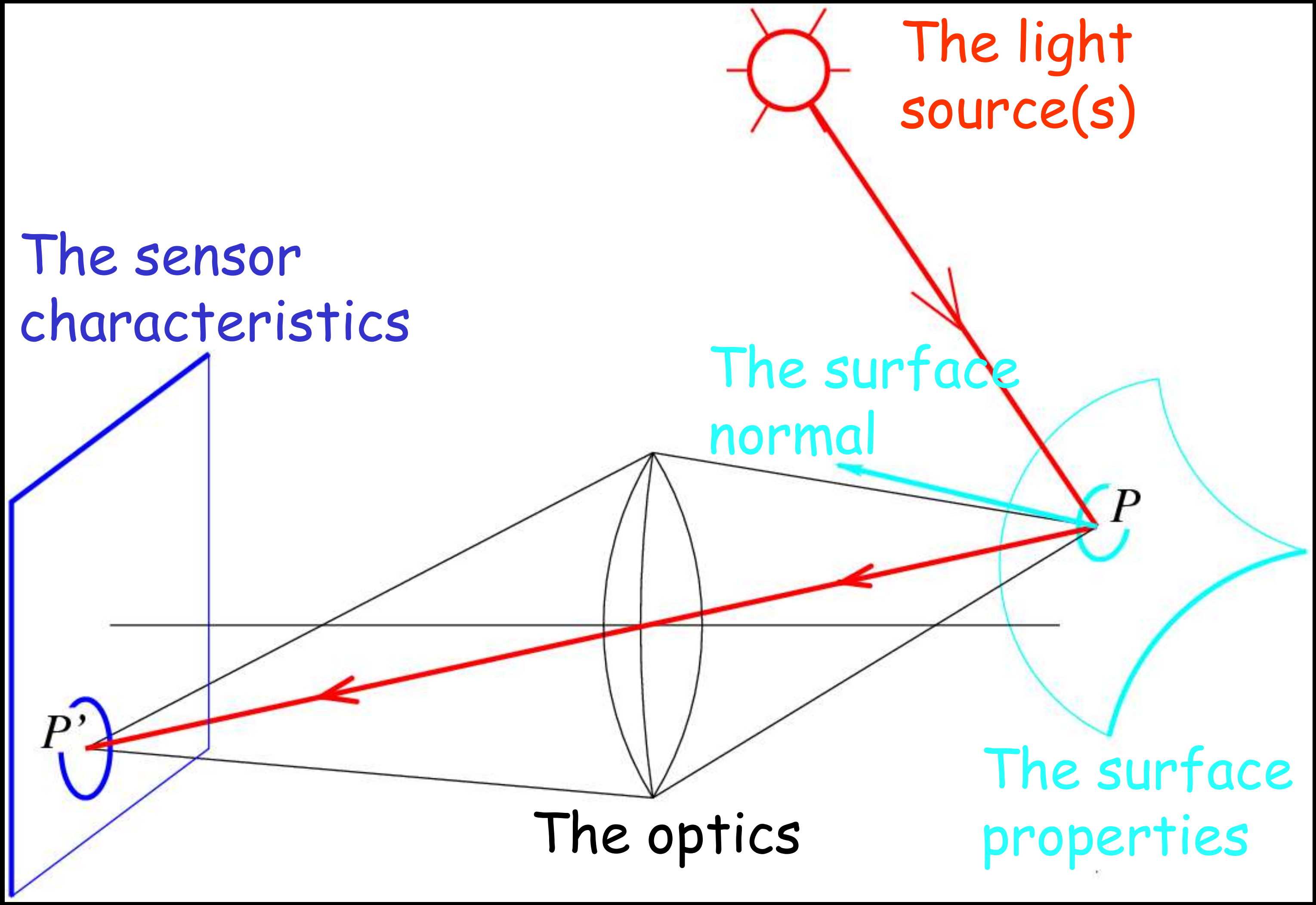
Photography
(Niepce, "La Table Servie," 1822)

Milestones:
- Daguerréotypes (1839)
- Photographic Film (Eastman, 1889)
- Cinema (Lumière brothers, 1895)
- Color Photography (Lumière brothers, again, 1908)
- Television (Baird, Farnsworth, Zworykin, 1920s)



CCD Devices (1970), etc.

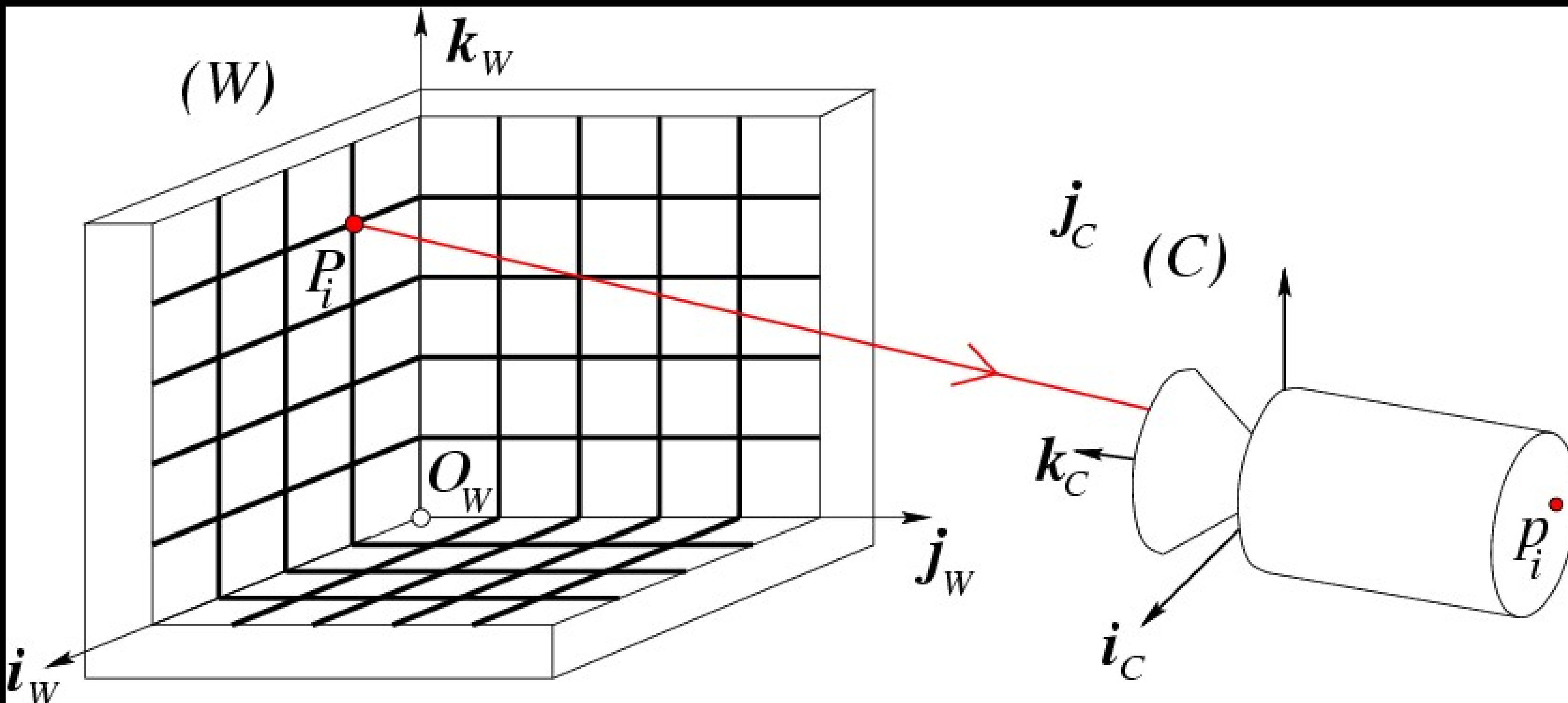
Image Formation: Radiometry



What determines the brightness of an image pixel?

Perspective Projection	$x' = f\frac{x}{z}$ $y' = f\frac{y}{z}$	x,y: World coordinates x',y': Image coordinates f: pinhole-to-retina distance
Weak-Perspective Projection (Affine)	$x' \approx -mx$ $y' \approx -my$ $\quad m = -\frac{f}{\bar{z}}$	x,y: World coordinates x',y': Image coordinates m: magnification
Orthographic Projection (Affine)	$x' \approx x$ $y' \approx y$	x,y: World coordinates x',y': Image coordinates
Common distortion model	$x'' = \frac{1}{\lambda}x'$ $y'' = \frac{1}{\lambda}y'$ $\lambda = 1 + k_1 r^2 + k_2 r^4 + \cdots$	x',y': Ideal image coordinates x'',y'': Actual image coordinates

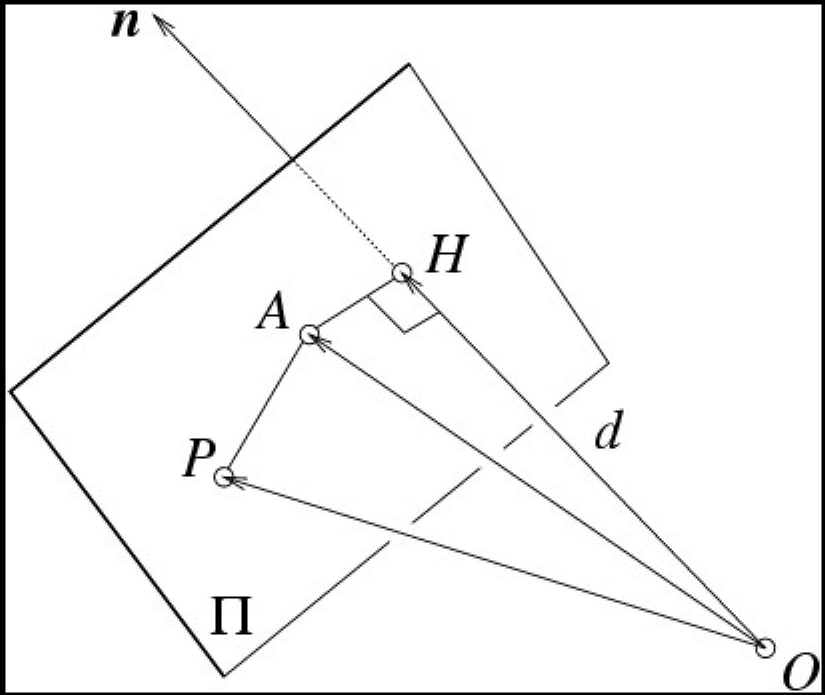
Quantitative Measurements and Calibration



Euclidean Geometry

Planes and homogeneous coordinates:

The translation vector between any two points in the plane is orthogonal to ***n***



$$\overrightarrow{AP}.\mathbf{n} = 0 \Leftrightarrow ax + by + cz - d = 0 \Leftrightarrow \mathbf{\Pi}.\mathbf{P} = 0$$

$$\text{where} \quad \mathbf{\Pi} = \begin{bmatrix} a \\ b \\ c \\ -d \end{bmatrix} \quad \text{and} \quad \mathbf{P} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

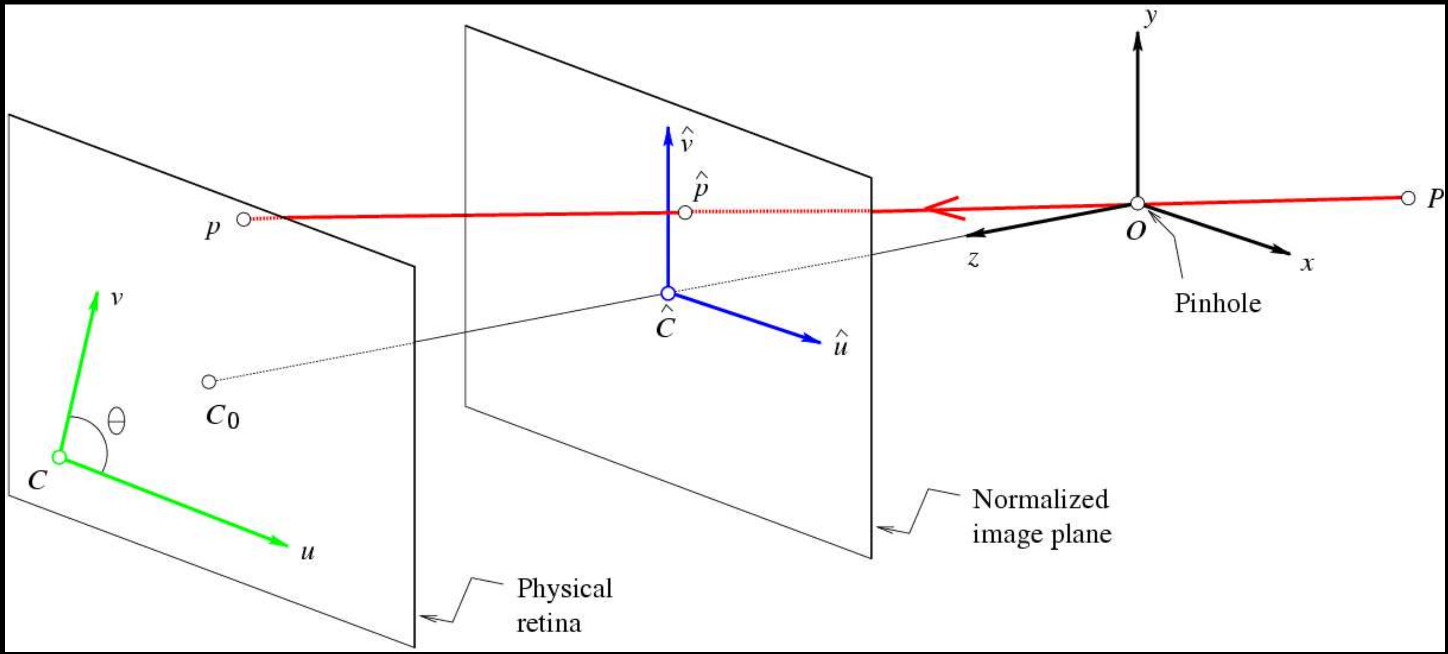
The intrinsic parameters of a camera

Units:

k, l : pixel/m

f : m

α, β : pixel



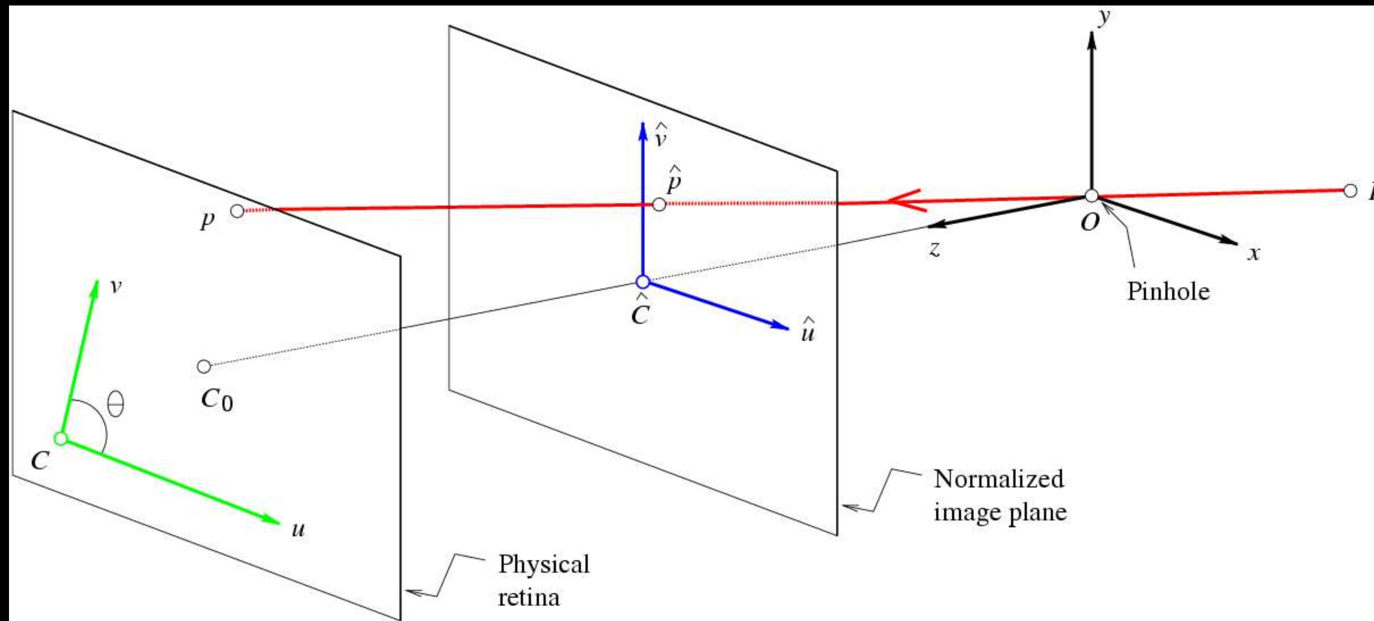
$$\begin{cases} \hat{u} = \dfrac{x}{z} \\ \hat{v} = \dfrac{y}{z} \end{cases} \iff \hat{\boldsymbol{p}} = \frac{1}{z} \begin{pmatrix} \mathrm{Id} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \boldsymbol{P} \\ 1 \end{pmatrix}$$

Normalized image coordinates

Physical image coordinates

$$\begin{cases} u = kf\dfrac{x}{z} \\ v = lf\dfrac{y}{z} \end{cases} \rightarrow \begin{cases} u = \alpha\dfrac{x}{z} + u_0 \\ v = \beta\dfrac{y}{z} + v_0 \end{cases} \rightarrow \begin{cases} u = \alpha\dfrac{x}{z} - \alpha \cot\theta \dfrac{y}{z} + u_0 \\ v = \dfrac{\beta}{\sin\theta}\dfrac{y}{z} + v_0 \end{cases}$$

The intrinsic parameters of a camera



Calibration matrix

$$\boldsymbol{p} = \mathcal{K}\hat{\boldsymbol{p}}, \quad \text{where} \quad \boldsymbol{p} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \quad \text{and} \quad \mathcal{K} \stackrel{\text{def}}{=} \begin{pmatrix} \alpha & -\alpha \cot\theta & u_0 \\ 0 & \dfrac{\beta}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

Homogeneous coordinates

The perspective projection equation

$$\boldsymbol{p} = \frac{1}{z}\mathcal{M}\boldsymbol{P}, \quad \text{where} \quad \mathcal{M} \stackrel{\text{def}}{=} (\mathcal{K} \quad \mathbf{0})$$

The Extrinsic Parameters of a Camera

- When the camera frame (C) is different from the world frame (W),

$$\begin{pmatrix} {}^C P \\ 1 \end{pmatrix} = \begin{pmatrix} {}^C_W \mathcal{R} & {}^C O_W \\ \mathbf{0}^T & 1 \end{pmatrix} \begin{pmatrix} {}^W P \\ 1 \end{pmatrix}.$$

- Thus,

$$\boxed{\boldsymbol{p} = \frac{1}{z} \mathcal{M} \boldsymbol{P},} \quad \text{where} \quad \begin{cases} \mathcal{M} = \mathcal{K} \, (\, \mathcal{R} \quad \boldsymbol{t} \,) \, , \\ \mathcal{R} = {}^C_W \mathcal{R}, \\ \boldsymbol{t} = {}^C O_W, \\ \boldsymbol{P} = \begin{pmatrix} {}^W P \\ 1 \end{pmatrix}. \end{cases}$$

- Note: z is *not* independent of $\mathcal{M}$ and $\boldsymbol{P}$:

$$\mathcal{M} = \begin{pmatrix} \boldsymbol{m}_1^T \\ \boldsymbol{m}_2^T \\ \boldsymbol{m}_3^T \end{pmatrix} \Longrightarrow z = \boldsymbol{m}_3 \cdot \boldsymbol{P}, \quad \text{or} \quad \begin{cases} u = \dfrac{\boldsymbol{m}_1 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}}, \\ v = \dfrac{\boldsymbol{m}_2 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}}. \end{cases}$$