#### Introduction to computer vision XIV

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Slides will be available after class at: https://mtrager.github.io/introCV-fall2019/

## Image categorization as supervised classification





Test image

Labelled training examples

## Image categorization as supervised classification



#### Convolutional neural networks (LeCun et al., 1998)



(Krizhevsky et al.'12)

#### Supervision: Where do the labels come from?

• A trend toward manually annotating the whole wide world with crowd sourcing



- Example: MS COCO (Lin et al., 2015)
  - 328K images of 91 object categories
  - 2.5M labelled instances

(Russell et al., 2008; Deng et al., 2009; Everingham et al., 2010; Xiao et al., 2010)

#### Outline

- > Weaker forms of supervision, e.g.,
  - > image-level labels
  - > existing meta data
- Not covered: Semi-supervised methods
  - with some labelled data
- > Totally unsupervised methods,
  - $\succ$  self-supervised  $\approx$  "free" labels
  - > and alternatives
- > Musings about parts, semantics, etc.

#### Using weaker supervision: Cosegmentation



(Lazebnik et al.'04; Rother et al.'06; Hochbaum & Singh'09; Joulin et al.'10) (Kim & Xing'11; Joulin et al.'12; Rubio et al.'12; Wang et al.'13)

#### Conventional supervised classification

$$\min_{f \in \mathcal{F}} \frac{1}{N} \sum_{n} \ell(z_n, f(\phi(x_n))) + \Omega(f)$$

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$$\min_{f \in \mathcal{F}} \frac{1}{N} \sum_{n} \ell(z_n, f(\phi(x_n))) + \Omega(f)$$

#### Discriminative clustering



(Xu et al., 2004; Bach & Harchaoui, 2007)



(Shi & Malik, 2000; Ng et al., 2001; Xu et al., 2004; Bach & Harchaoui, 2007)

#### Multi-class cosegmentation (Joulin et al., CVPR'12)

$$\min_{\substack{y \in \{0,1\}^{N \times K} \\ y \mathbbm{1}_K = \mathbbm{1}_N}} \left[ \min_{\substack{A \in \mathbbm^{d \times K} \\ b \in \mathbbm^K}} E_U(y, A, b) \right] + E_B(y) - H(y)$$

Optimization:

- Relax to continuous problem
- EM/block-coordinate descent procedure with quasi-Newton and projected gradient descent for the two steps, initialized with quadratic approximation
- Round up the solution

Missing: no foreground model (Rother el al., 2006)

#### Multi-class cosegmentation results



#### (Joulin et al., CVPR'12)

#### Naming the characters of TV series



(Sivic, Everingham, Zisserman, 2009)

#### TV series come with their own metadata



(Sivic, Everingham, Zisserman, 2009)

As the headwaiter takes them to a table they pass by the piano, and the woman looks at Sam. Sam, with a conscious effort, keeps his eyes on the keyboard as they go past. The headwaiter seats Ilsa...

#### Videos (often) come with their own metadata!

As the headwaiter takes them to a table they pass by the piano, and the woman looks at Sam. Sam, with a conscious effort, keeps his eyes on the keyboard as they go past. The headwaiter seats Ilsa...



#### Scripts as a source of supervision



(Laptev et al., 2008; Sivic et al., 2009; Duchenne et al., 2009)

#### Automated temporal action localization

Input:

- Action type, e.g.
  "Person opens door"
- Videos + aligned scripts

#### Output: temporal action clusters

Jane jumps up and opens the door ...
 Carolyn opens the front door ...
 Jane opens her bedroom door ...



(Duchenne, Laptev, Sivic, Bach, Ponce, 2009)

#### Temporal localization as classification

#### Feature space



#### Video space



#### Negative samples



Random video clips: lots of them, very low chance to be positives

#### A latent SVM model for temporal localization

(Felzenszwalb, McAllester, Ramanan, 2008)

#### Feature space

$$\min_{w,b} C_{+} \sum_{i=1}^{M} \max\{0, 1 - \max_{f} w^{\top} \Phi(c_{i}[f]) - b \\ + C_{-} \sum_{i=1}^{P} \max\{0, 1 + w^{\top} \Phi(x_{i}^{-}) + b\} + \|w\|^{2}$$

Optimization: Block-coordinate descent

1. Exhaustive search for f

2. SVM training for w, bThis is an instance of discriminative clustering

#### Clustering results on "Coffee and cigarettes"





#### **Automatic Annotation of Human Actions in Video**

#### **ICCV 2009 DEMO**

O.Duchenne, I.Laptev, J.Sivic, F.Bach and J.Ponce

Temporal detection of actions OpenDoor and SitDown in episodes of The Graduate, The Crying Game, Living in Oblivion

#### Exploiting temporal constraints



#### Action labeling under ordering constraints (Bojanowski et al., ECCV'14)



Changing the representation

#### $a, m \rightarrow Z$



#### Action labeling under ordering constraints



- Minimize a convex quadratic function over a large discrete domain z
- Relaxed problem: minimize instead over  $\overline{Z}$ =conv(Z), then round up
- Difficulty: Z (and thus  $\overline{Z}$ ) are defined by complex implicit constraints
- Frank-Wolfe to the rescue!

#### The Frank-Wolfe algorithm (1956)



Repeat until convergence :

- Replace the cost surface by its tangent plane, and minimize over  $\overline{Z}$
- $Z_{k+1} = (1-\gamma) Z_k + Z^*$
- No need for a projection step, converges to global minimum
- DP can be used to minimize linear functions over z and thus  $\overline{z}$
- DP can also be used for rounding

#### Temporal action localization

### Clip number 0101

(Bojanowski et al., ECCV'14)

#### Learning from narrated instructional videos (Alayrac et al., CVPR'16, PAMI'17)

We can get two hands on it and we can exert some real leverage

#### Making assembly plans



Automatically produce a sequence of instructions from narrated videos

#### 



Start by loosening each bolt. Then locate the jack and lift the car. Now you can remove the bolts and then the wheel.

#### 

 $\emptyset$  First undo the nuts. Once that done, you can jack the car. Then withdraw the nuts completely so that you can remove the flat tire.

#### 



Start by loosening each bolt. Then locate the jack and lift the car. Now you can remove the bolts and then the wheel.

First undo the nuts. Once that done, you can jack the car. Then withdraw the nuts completely

#### Output:

• Sequence of main steps

so that you can remove the flat tire.

1. Loosen nuts

2. Jack the car

3. Remove the flat tire

## Start by loosening each bolt. Then locate the jack and lift the car. Now you can remove the bolts and then the wheel. Start by loosening each bolt. Then locate the jack and lift the car. Now you can remove the bolts and then the wheel.

First undo the nuts. Once that done, you can jack the car. Then withdraw the nuts completely so that you can remove the flat tire.

#### Output:

- Sequence of main steps
- Visual and textual models of the steps

1. Loosen nuts

2. Jack the car

3. Remove the flat tire

# First undo the nuts. Once that done, you can jack the car. Then withdraw the nuts completely so that you can remove the flat tire.

#### Output:

- Sequence of main steps
- Visual and textual models of the steps
- Temporal localization of the steps

Loosen nuts
 Jack the car
 Remove the flat tire

#### Text alignment in multiple sequences

- Narrations are first processed into sequence of direct object relations (dobj)
  - Ex: "Let's now jack the car" -> dobj = [jack car]
- Similarity scores from Wordnet
  - Ex: undo bolt  $\approx$  loosen nut, jack car  $\neq$  remove wheel

| Video 1      | Video 2     | Video 3       | Video 4   |
|--------------|-------------|---------------|-----------|
| jack car     | loosen nut  | loosen nut    | undo bolt |
| remove wheel | raise car   | jack car      | lift car  |
|              | remove tire | unscrew nut   | lower car |
|              | lower jack  | withdraw tire |           |
| Video 1      | Video 2     | Video 3       | Video 4   |
|--------------|-------------|---------------|-----------|
| jack car     | loosen nut  | loosen nut    | undo bolt |
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|              | lower jack  | withdraw tire |           |

| Video 1      | Video 2     | Video 3       | Video 4   |
|--------------|-------------|---------------|-----------|
| $\boxtimes$  | loosen nut  | loosen nut    | undo bolt |
| jack car     | raise car   | jack car      | lift car  |
| remove wheel | remove tire | unscrew nut   | lower car |
|              | lower jack  | withdraw tire |           |

Video 1

Video 2

Video 3

Video 4

ø jack car remove wheel loosen nut raise car remove tire lower jack loosen nut jack car unscrew nut withdraw tire

undo bolt lift car lower car





# We seek to minimize the sum of pairwise costs:



 $C = c(\emptyset, \text{`loosen nut'}) + \dots + c(\emptyset, \text{`undo bolt'})$  $+ \dots + c(\text{`jack car'}, \text{`raise car'})$  $+ \dots + c(\emptyset, \text{`unscrew nut'}) + \dots$ 



[Wang and Jiang 1994, Higgins and Sharp, 1988]

Rewrite as an integer quadratic program

$$\min_{U} \operatorname{Tr}(U^T B U), \text{subject to } U \in \bar{\mathcal{U}}$$

- Solve relaxed problem with Frank-Wolfe
- Round up the solution

[Wang and Jiang 1994, Higgins and Sharp, 1988]





.. and then use method similar to previous one for temporal localization



# Activity discovery from images and words

#### Make kimchi fried rice



"I'm going to start off by <mark>chopping</mark> up an <mark>onion</mark>.

Get your pan on a nice high heat and add some <mark>oil</mark>...

...and just <mark>stir</mark> this through...

... You want to fry the rice now mixing every now and then..."

#### Make Kerala fish curry

"...next you're going to take a full <mark>onion</mark>...

...So make sure you stir the onions around...

...you're also going to add one cup of water to the pan..."

(Alakuijala, Mairal, Ponce, Schmid, 2019)

# Activity discovery from images and words



(Alakuijala, Mairal, Ponce, Schmid, 2019)

#### Discovered ingredients of "performing CPR"





art and the stop of the second of the second

recheck minutesUSE finger about the continue cycle give information commence cpr little battraction find one establish cpr get help continue bit recever bit keep victim need bit

tilt forehead keep chin tilt finger tilt head look chest place mask use head lock elbow check airway push ear reposition airway reopen airway open airway have partner

have patient say nose have patient say nose tap shoulder repeat cycle have someone do cprkeep blood find center approach casualty

establish responsiveness interlock finger check circulation help heart contact skin have mouthguard avoid contact say help heart bill bystander have person push help hear have person keep interruption assess scene have blood administer breath

#### How much supervision do we really need? (Cho et al., CVPR'15)



Object detection (Leibe et al.'08; Felzenszwalb et al.'10; Girshick et al.'14) Weakly supervised localization (Chum'07;Pandey'11;Desaelers'12;Siva'12;Shi'13;Cinbis'14;Wang'14) Co-segmentation/localization (Rother'06;Russell'06;Joulin'10;Kim'11;Vicente'11;Joulin'14;Tang'14) Unsupervised discovery (Grauman & Darrell'05; Sivic et al'05,08; Kim et al.'05,09)

#### Using context for self supervision Ex: Word2vec (Mikolov et al., 2013): $w \rightarrow u(w) \in \mathbb{R}^d$

- $u(\text{Paris}) \approx u(\text{France}) + [u(\text{Berlin}) u(\text{Germany})]$
- Analogies as "linear algebra"



Modeling contextual info with co-occurrence statistics

$$\max_{u,v} \frac{1}{T} \sum_{t=1}^{T} \sum_{c \in N_t} \left( \log \sigma[u(w_t) \cdot v(c)] + \sum_{k=1}^{K} \log \sigma[-u(w_t) \cdot v(w_{\text{random}})] \right)$$

#### Example: Unsupervised Visual Representation Learning by Context Prediction (Doersch, Gupta, Efros, 2016)

#### Example:







#### Question 1:



#### Question 2:



# Retrieved nearest neighbors



#### Unsupervised feature learning (Bojanowski & Joulin, ICML'17)



# Retrieved nearest neighbors





















(Cho, Kwak, Schmid, Ponce, 2015)









#### Matching







(Russell et al.'06; Cho et al.'10; Deselaers et al.'10; Rubinstein & Joulin'13; Rubio et al.'13)

#### Finding parts and objects among region candidates





1000 to 4000 candidates per object

Here: Region proposals (Manen et al.'13, Uijlings et al.'13) and HOG descriptors (Dalal & Triggs'05)

## Finding parts and objects among region candidates





1000 to 4000 candidates per object

Caveat: These region proposals are supervised

# Matching model - Probabilistic Hough matching match data configuration $P(m | d) = \sum_{c} P(m | c, d) P(c | d)$

two regions region proposals position+scale

$$m = [r, r']$$





• Bayesian model

 $P(m \mid d) = \sum_{c} P(m \mid c) P(c \mid d)$  $= P(m_{a}) \sum_{c} P(m_{g} \mid c) P(c \mid d)$ 

Bayesian model

 $P(m \mid d) = \sum_{c} P(m \mid c) P(c \mid d)$  $= P(m_{a}) \sum_{c} P(m_{g} \mid c) P(c \mid d)$ 

• Probabilistic Hough transform

$$P(c \mid d) \approx H(c \mid d) = \sum_{m \in d} P(m \mid c)$$
$$= \sum_{m \in d} P(m_a) P(m_g \mid c)$$

(Hough'59; Ballard'81; Stephens'91; Leibe et al.'04; Maji & Malik'09; Barinova et al.'12)

• Bayesian model

$$P(m \mid d) = \sum_{c} P(m \mid c) P(c \mid d)$$
  
=  $P(m_a) \sum_{c} P(m_g \mid c) P(c \mid d)$ 

• Probabilistic Hough transform

$$P(c \mid d) \approx H(c \mid d) = \sum_{m \in d} P(m \mid c)$$
  
=  $\sum_{m \in d} P(m_a) P(m_g \mid c)$ 

Region confidence

$$C(r' \mid d) = \max_{r''} P(r' \leftrightarrow r'' \mid d)$$









• Bayesian model

$$P(m \mid d) = \sum_{c} P(m \mid c) P(c \mid d)$$
  
=  $P(m_a) \sum_{c} P(m_g \mid c) P(c \mid d)$ 

• Probabilistic Hough transform

$$P(c \mid d) \approx H(c \mid d) = \sum_{m \in d} P(m \mid c)$$
  
=  $\sum_{m \in d} P(m_a) P(m_g \mid c)$ 

Two images -> multiple images

$$C_{d'}(r') = \sum_{d''} C(r' | [d', d''])$$



#### Stand-out scoring of part hierarchies



- Object regions should contain
  - more foreground than part regions
  - less background than larger regions
#### Stand-out scoring of part hierarchies



- Object regions should contain
  - more foreground than part regions
  - less background than larger regions

• 
$$S_d(r) = C_d(r) - \max_{r' \supset r} C_d(r')$$





















#### Initialize



Retrieve 10 nearest neighbors (Oliva & Torralba'06)



#### Match



Localize















#### Localize



Retrieve using top 20 confidence scores, etc.

## Localization improvement over iterations



#### After 1 iteration



#### After 3 iterations





## Retrieval improvement over iterations



### Pascal'07 results (Cho et al., CVPR'15)

#### CorLoc - separate classes

| Method                 | Data used | aero | bicy | bird | boa  | bot  | bus  | car  | cat  | cha  | cow  | dtab | dog  | hors | mbik | pers | plnt | she  | sofa | trai | tv   | Av.  |
|------------------------|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| Pandey & Lazebnik [26] | P + N     | 50.9 | 56.7 | -    | 10.6 | 0    | 56.6 | -    | -    | 2.5  | -    | 14.3 | -    | 50.0 | 53.5 | 11.2 | 5.0  | -    | 34.9 | 33.0 | 40.6 | -    |
| Siva & Xiang [36]      | P + A     | 42.4 | 46.5 | 18.2 | 8.8  | 2.9  | 40.9 | 73.2 | 44.8 | 5.4  | 30.5 | 19.0 | 34.0 | 48.8 | 65.3 | 8.2  | 9.4  | 16.7 | 32.3 | 54.8 | 5.5  | 30.4 |
| Siva et al. [34]       | P + N     | 45.8 | 21.8 | 30.9 | 20.4 | 5.3  | 37.6 | 40.8 | 51.6 | 7.0  | 29.8 | 27.5 | 41.3 | 41.8 | 47.3 | 24.1 | 12.2 | 28.1 | 32.8 | 48.7 | 9.4  | 30.2 |
| Shi et al. [33]        | P + N     | 67.3 | 54.4 | 34.3 | 17.8 | 1.3  | 46.6 | 60.7 | 68.9 | 2.5  | 32.4 | 16.2 | 58.9 | 51.5 | 64.6 | 18.2 | 3.1  | 20.9 | 34.7 | 63.4 | 5.9  | 36.2 |
| Cinbis et al. [6]      | P + N     | 56.6 | 58.3 | 28.4 | 20.7 | 6.8  | 54.9 | 69.1 | 20.8 | 9.2  | 50.5 | 10.2 | 29.0 | 58.0 | 64.9 | 36.7 | 18.7 | 56.5 | 13.2 | 54.9 | 59.4 | 38.8 |
| Wang et al. [42]       | P + N + A | 80.1 | 63.9 | 51.5 | 14.9 | 21.0 | 55.7 | 74.2 | 43.5 | 26.2 | 53.4 | 16.3 | 56.7 | 58.3 | 69.5 | 14.1 | 38.3 | 58.8 | 47.2 | 49.1 | 60.9 | 48.5 |
| Joulin et al. [18]     | P         | -    | -    | -    | -    | -    | -    | -    | -    | -    | -    | -    | -    | -    | -    | -    | -    | -    | -    | -    | -    | 24.6 |
| Ours                   | P         | 50.3 | 42.8 | 30.0 | 18.5 | 4.0  | 62.3 | 64.5 | 42.5 | 8.6  | 49.0 | 12.2 | 44.0 | 64.1 | 57.2 | 15.3 | 9.4  | 30.9 | 34.0 | 61.6 | 31.5 | 36.6 |

#### CorLoc and CorRet - mixed classes Uses pre-trained CNN features

| Evaluation metric | aero | bicy | bird | boa  | bot  | bus  | car  | cat  | cha  | cow  | dtab | dog  | hors | mbik | pers | plnt | she  | sofa | trai | tv   | Av.  | any  |
|-------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| CorLoc            | 40.4 | 32.8 | 28.8 | 22.7 | 2.8  | 48.4 | 58.7 | 41.0 | 9.8  | 32.0 | 10.2 | 41.9 | 51.9 | 43.3 | 13.0 | 10.6 | 32.4 | 30.2 | 52.7 | 21.8 | 31.3 | 37.6 |
| CorRet            | 51.1 | 45.3 | 12.7 | 12.1 | 11.4 | 21.2 | 61.9 | 11.6 | 19.2 | 9.70 | 3.9  | 17.2 | 29.6 | 34.0 | 43.7 | 10.2 | 8.1  | 9.9  | 23.7 | 27.3 | 23.2 | 36.6 |

#### Examples - mixed classes

Successes

#### Failures



### Unsupervised object discovery in multiple videos



(Suha, Cho, Laptev, Ponce, Schmid, 2015)

#### aeroplane-0004-029

- Object colocalization per class
  - Unsupervised object discovery



#### Copyright © Simon Lowe

#### bird-0004-016

- Object colocalization per class
- Unsupervised object discovery



#### (Suha, Cho, Laptev, Ponce, Schmid, 2015)

## 45 clips selected manually from the Bourne trilogy

# **Discovering Cars** from movie clips



About 90mn (excluding preprocessing) on a 12-core 1.2GHz machine

#### 44 clips selected manually from two movies

## **Discovering Animals** from movie clips



About 90mn (excluding preprocessing) on a 12-core 1.2GHz machine

# Going further

## Unsupervised object discovery as optimization



(Vo, Bach, Cho, Han, LeCun, Perez, Ponce, CVPR'19)



## w/o CO: greedy combinatorial search w CO: use gradient ascent

| Method                    | OD                               | VOC_6x2                          | VOC_all                          |
|---------------------------|----------------------------------|----------------------------------|----------------------------------|
| Cho et al.                | -                                | -                                | 37.6                             |
| Cho et al., our execution | 82.2                             | 55.9                             | 37.5                             |
| w/o CO                    | $\textbf{83.0} \pm \textbf{0.4}$ | $\textbf{60.2} \pm \textbf{0.4}$ | $\textbf{39.8} \pm \textbf{0.2}$ |
| w CO                      | $80.8\pm0.5$                     | $59.3\pm0.4$                     | $38.5\pm0.2$                     |

# Unsupervised region proposals and large-scale object discovery using features trained on an auxiliary task



(Vo, Perez, Ponce, 2019)

Insight: sum of feature map values provide good saliency maps (Wei et al., 2017), and thus a good basis for region proposals Insight 2: Use two interpretations of the graph: - Proxy for the true structure. Run algorithm on small subgroups of images with v=50 to find promising proposals - True structure. Run the algorithm with the selected proposals and v=5 on the whole image collection

#### Small-scale CorLoc results

Proposal comparison

| Method   | Features                | OD                                       | VOC_6x2                                    | VOC_all                                    |
|--|-------------------------|--|--|--|
| Cho et al.<br>Vo et al. RP                               | WHO<br>WHO              | $82.2$ $\underline{82.3 \pm 0.3}$        | $55.9 \\ 62.5 \pm 0.6$                     | $37.6 \\ 40.7 \pm 0.2$                     |
| Wei <i>et al.</i> [33]<br>Wei <i>et al.</i> [34]<br>Ours | VGG16<br>VGG16<br>VGG16 | 75.8<br>73.5<br><b>87.5</b> ± <b>0.3</b> | $57.9 \\ \underline{66.2} \\ 70.9 \pm 0.3$ | $39.8 \\ \underline{41.9} \\ 48.6 \pm 0.1$ |

| Region proposals      | OD                               | VOC_6x2                          | VOC_all                          |
|-----------------------|----------------------------------|----------------------------------|----------------------------------|
| Edgeboxes [37]        | $81.4 \pm 0.3$                   | $55.2 \pm 0.3$                   | $32.6\pm0.1$                     |
| Selective search [30] | $81.3 \pm 0.3$                   | $57.8\pm0.2$                     | $33.0\pm0.1$                     |
| Randomized Prim [22]  | $82.5 \pm 0.1$                   | $70.6\pm0.4$                     | $44.5\pm0.1$                     |
| Ours                  | $\textbf{87.5} \pm \textbf{0.3}$ | $\textbf{70.9} \pm \textbf{0.3}$ | $\textbf{48.6} \pm \textbf{0.1}$ |

#### Large-scale CorLoc results

| Method   | VOC_all                         | VOC12   | COCO_20k                         |
|--|---------------------------------|---|----------------------------------|
| Wei <i>et al.</i> [33]<br>Wei <i>et al.</i> [34] | 43.4<br>43.4                    | 46.2<br>46.3  | 38.6<br>40.5                     |
| Baseline 1                                       | $43.3\pm0.2$                    | $40.1 \pm 0.1$  | $45.0\pm0.1$                     |
| Baseline 2<br>Ours                               | $ 48.6 \pm 0.1   46.5 \pm 0.1 $ | $\begin{array}{l} \textbf{49.3} \pm \textbf{0.1} \\ \textbf{46.2} \pm \textbf{0.1} \end{array}$ | $\textbf{47.3} \pm \textbf{0.1}$ |

#### Large-scale CorRet results

| Dataset          | VOC_all   | VOC12                            | COCO_20k  |
|------------------|---|----------------------------------|---|
| Baseline<br>Ours | $\begin{array}{c} 50.7\\ \textbf{61.3}\pm\textbf{0.0}\end{array}$ | 57.5<br><b>64.7</b> ± <b>0.0</b> | $\begin{array}{c} 36.8 \\ \textbf{40.1} \pm \textbf{0.0} \end{array}$ |



## Beyond block diagrams and pattern recognition

Deep learning (LeCun et al.'98)



Tunable algorithms (Eboli et al.'19, Lecouat et al.'19)

function  $x = \text{LCHQS}(y, k_0, K, \theta, \nu)$  $x = y; \mu = 0;$ for t = 0 : T - 1 do  $\widehat{z} = [y; \sqrt{\mu} \varphi^{\theta}_{\lambda/\mu} (K \star x)];$  $\widehat{K} = [k_0; \sqrt{\mu K}];$  $C = \operatorname{argmin}_{C} ||\delta - C \star \widehat{K}||_{F}^{2} + \rho \sum_{i=0}^{n} ||c_{i}||_{F}^{2};$  $x = \operatorname{CPCR}(\widehat{K}, \widehat{z}, \psi^{\nu}(C), x);$  $\mu = \mu + \delta_t;$ end for end function function  $x = CPCR(A, b, C, x_0)$  $x = x_0;$ for u = 0 : U - 1 do  $x = x - C \star (A \star x - b);$ end for end function

